FE - SEM II (CBGS)
Applied maths - II

May 2014
15/5/14

(REVISED COURSE) QP Code : NP-17762

(3 Hours) [Total Marks : 80

N.B. : (1) Question **No.1** is **compulsory.**
(2) Attempt any **three** question from remaining **five** questions.

1. (a) Evaluate $\int_0^1 \sqrt{(\sqrt{x}-x)}\,dx$ **3**

(b) Solve $[D^4 - 4D^3 + 8D^2 - 8D + 4]\,y = 0$ **3**

(c) Prove that $(1+\Delta)(1-\nabla) = 1$ **3**

(d) Change to polar co-ordinate and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dy\,dx$ **3**

(e) Solve $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$ **4**

(f) Evaluate $\int_0^a \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dy\,dx$ **4**

2. (a) Solve $xy(1+xy^2)\frac{dy}{dx} = 1$ **6**

(b) Change the order of integration and evaluate $\int_0^\infty \int_0^x x e^{-x^2/y}\,dy\,dx$ **6**

(c) Evaluate $\int_0^{\pi/2} \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$ and show that **8**

$$\int_0^{\pi/2} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2} = \frac{\pi}{4ab}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

3. (a) Evaluate $\iiint x^2\,yz\,dx\,dy\,dz$ throughout the volume bounded by $x=0, y=0, z=0$, $x+y+z=1$. **6**

(b) Find the area bounded by parabola $y^2 = 4x$ and the line $y = 2x - 4$. **6**

(c) Use the method of variation of parameter to solve **8**

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sec^2 x\,(1+2\tan x)$$

4. (a) Find the total length of the loop of the curve $9y^2 = (x+7)(x+4)^2$. **6**

   (b) Solve $\dfrac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x - \cos 2x$ **6**

   (c) Apply Runge-kutta method of fourth order to find an approximate value of y at $x = 0.2$ if $\dfrac{dy}{dx} = x + y^2$ give that $y = 1$, when $x = 0$ in step of $h = 0.1$. **8**

5. (a) Solve $y(x^2y + e^x)\,dx - e^x\,dy = 0$. **6**

   (b) Using Taylor's series method solve $\dfrac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$ and hence find $y(0.2)$ and $y(0.4)$. **6**

   (c) Compute the value of the definite integral $\int_{0.2}^{1.4}(\sin x - \log_e x + e^x)\,dx$, by **8**

   (i) Trapezoidal Rule
   (ii) Simpson's one third Rule
   (iii) Simpson's three-eigth Rule.

6. (a) The motion of a particle is given by $\dfrac{d^2x}{dt^2} = -k^2x - 2h\dfrac{dx}{dt}$, solve the equation when $h = 5, k = 4$ taking $x = 0$, $v = v_0$ at $t = 0$. Show that the time of maximum displacement is independent of the initial velocity. **6**

   (b) Evaluate $\iint (x^2 + y^2)\,dx\,dy$ over the area of triangle whose vertices are (0, 0), (1, 0), (1, 2). **6**

   (c) Find the volume bounded by $y^2 = x$, $x^2 = y$ and the planes $z = 0$ and $x + y + z = 1$. **8**

Con. 8055-13. **GX-10120**

(Revised course)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. 1 is **compulsory**.
(2) Answer any **three** questions from question nos. 2 to 6.
(3) **Figures** to the **right** indicate **full** marks.
(4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^{\infty} x^2\, 7^{-4x^2}\, dx$ 3

(b) Solve $(D^4+4)y=0$ 3

(c) Prove that $E\,\nabla=\Delta=\nabla E$ 3

(d) Solve $(x+2y^3)\,\frac{dy}{dx}=y.$ 3

(e) Evaluate $\iint_R r^3\, dr d\theta$ over the region between the circles $r=2\sin\theta$, $r=4\sin\theta$. 4

(f) Evaluate $\int_0^1\int_y^{\sqrt{y}} \frac{x}{(1-y)\sqrt{y-x^2}}\, dydx$ 4

2. (a) Solve :- $(x^3y^4+x^2y^3+xy^2+y)\,dx+(x^4y^3-x^3y^2-x^2y+x)dy=0$ 6

(b) Change the order of integral and hence evaluate $\int_0^5\int_{2-x}^{2+x} dxdy$ 6

(c) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\, dx \int_0^1 \frac{dx}{\sqrt{1+x^4}}=\frac{\pi}{4\sqrt{2}}$ 8

3. (a) Evaluate $\int_0^1\int_{y^2}^1\int_0^{1-x} x\, dxdydz.$ 6

(b) Find the area of one loop of the lemniscate $r^2=a^2\,.\cos 2\theta$ 6

(c) Solve $(D^3+2D^2+D)y=x^2e^{3x}+\sin^2 x+2^x.$ 8

4. (a) Show that the length of arc of the parabola $y^2=4ax$ cut off by the line $3y=8x$ is $a\left(\log 2+\frac{15}{16}\right)$ 6

(b) Using the method of variation of parameters solve $\frac{d^2y}{dx^2}+4y=\tan 2x.$ 6

(c) Compute $y(0.2)$ given $\frac{dy}{dx}+y+xy^2=0$, $y(0)=1$ by taking $h=0.1$ using Runge-Kutta method of fourth order correct to 4 decimals. 8

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5. (a) Solve $\frac{dy}{dx}+x(x+y)=x^3(x+y)^3-1$. 6

(b) Solve $\frac{dy}{dx}-2y=3e^X, y(0)=0$ using Taylor series method. Find approximate value of y for x = 1 and 1.1. 6

(c) Evaluate $\int_0^6 \frac{dx}{1+x}$ using 8

(i) Trapezoidal rule

(ii) Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule and

(iii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

Compare result with exact values.

6. (a) The current in a circuit containing an inductance L, registance R and voltage E sin wt is given by 6

$$L\frac{di}{dt}+Ri=E\sin wt$$

If i = 0 at t = 0, find i.

(b) Evaluate $\iint_R e^{2x-3y}\, dxdy$ over the triangle bounded by $x+y=1,\ x=1, y=1$. 6

(c) (i) Find the volume of solid bounded by the surfaces $y^2=4ax$, $x^2=4ay$ and the planes Z = 0, Z = 3. 4

(ii) Change to polar co-ordinates and evaluate 4

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$$

F.E Sem II (R)
App. Maths - II

10/6/2013

wk Feb 2013-(e) 97

Con. 6887-13. (REVISED COURSE) GS-5427

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **three** questions from Question Nos. **2** to **6**.
(3) **Figures** to the **right** indicate **full** marks.
(4) **Programmable** calculators are not **allowed**.

1. (a) Evaluate $\int_0^1 (x \log x)^4 \, dx$. **3**

(b) Solve $(D^2 - 1)(D - 1)^2 y = 0$. **3**

(c) prove that $E = 1 + \Delta = e^{hD}$. **3**

(d) Solve $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$ **3**

(e) Change into Polar co-ordinates and Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\, dy\, dx$. **4**

(f) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx\,dy}{1+x^2+y^2}$ **4**

2. (a) Solve $(x^3 y^3 - xy)\, dy = dx$. **6**

(b) Change the order of Integration and Evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dy\, dx$. **6**

(c) (i) P.T. $\int_0^{\pi/2} \tan^n x\, dx = \frac{\pi}{2} \sec\left[\frac{n\pi}{2}\right]$. **4**

(ii) Evaluate $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\, dx$, $a > 0$ **4**

3. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\,dy\,dx}{(1+x+y+z)^3}$. **6**

(b) Find the area using Double integration where the regionof integration is bounded by the curves $9xy = 4$ and $2x + y = 2$. **6**

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(c) (i) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log y)$. 4

(ii) Solve the equation by method of variation of parameters 4

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}.$$

4. (a) Show that for the parabola $r = \frac{2a}{1+\cos\theta}$ for $\theta = 0$ to $\frac{\pi}{2}$ is $a\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$. 6

(b) Solve $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x - \cos 2x$. 6

(c) Apply Runge-Kutta method of fourth order to find an approximation value of y at $x = 0{\cdot}2$ if $\frac{dy}{dx} = x+y^2$ given $y = 1$ when $x = 0$ in steps of $h = 0{\cdot}1$. 8

5. (a) Solve $(2xy^4e^y + 2xy^3 + y)\,dx + [x^2y^4e^y - x^2y^2 - 3x]\,dy = 0$. 6

(b) Solve $\frac{dy}{dx} = 2x+y$ with initial conditions $x_0 = 0$, $y_0 = 0$ by Taylor's method obtain y as series in powers of x. 6
Find approximation value of y for $x = 0{\cdot}2, 0{\cdot}4$. Compare your result with exact values.

(c) Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by : 8

(i) Trapizoidal method (ii) Simpson's $\frac{1}{3}^{rd}$ method and (iii) Simpsons $\frac{3}{8}^{th}$ method. Compare result with exact values.

6. (a) In a circuit containing inductance L, resistance R and voltage E. The current i is given by 6

$L\frac{di}{dt} + Ri = E$. Find current i at time t if $t = 0$, $i = 0$ and L, R, E are constants.

(b) Evaluate $\iint_R xy\,dx\,dy$ where R is the region bounded by $x^2 + y^2 - 2x = 0$, $y = x$ and $y^2 = 2x$. 6

(c) (i) Find volume of tetrahedron bounded by plane $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$. 4

(ii) Find volume bounded by cone $z^2 = x^2 + y^2$ and Paraboloid $z = x^2 + y^2$. 4