3/12/13

FE · SEM-I (CBGS) DEC 2013

V-A4-II-Ex-13-F-11

A.M. I

Con. 7380-13. **GX-10003**

(REVISED COURSE)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **three** from the **remaining**.

1. (a) If $\alpha + i\beta = \tanh\left(\chi + i\frac{\pi}{4}\right)$, prove that $\alpha^2 + \beta^2 = 1$. **3**

(b) If $u = x^2y + e^{xy^2}$ show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **3**

(c) If $u = 1 - x$, $v = x(1 - y)$, $w = xy(1 - z)$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = x^2y$. $-x^2y$ **3**

(d) Prove that $\log(1 - x + x^2) = -x + \frac{x^2}{2} + \frac{2x^3}{3} - - - -$ **3**

(e) Express the relation in $\alpha, \beta, \gamma, \delta$ for which $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary. **4**

(f) Find n^{th} derivative of $2^x\cos^2 x \sin x$. **4**

$2n\pi/3$

2. (a) $\cancel{Z}^3$ $z^3 = (z + 1)^3$, then show that $z = \frac{-1}{2} + \frac{i}{2}\cot\frac{\theta}{2}$ where $\theta = 20\frac{\pi}{3}$. **6**

(b) Find the non-singular matrices P and Q such that PAQ is in Normal Form. Also fiind rank of A. **6**

$$A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$$

(c) State and Prove Euler's theorem for homogeneous functions in two variables and hence find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ for **8**

$u = e^{x+y} + \log(x^3 + y^3 - x^2y - xy^2)$

λz?

3. (a) For what values of λ the system of equations have ~~*~~ non-trivial solution ? Obtain the solution for real values of λ where $3x + y - \lambda x = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y - \lambda 2 = 0$. **6**

(b) Find the stationary values of $\sin x \sin y \sin(x + y)$. **6**

(c) If $\cos(x + iy)\cos(u + iv) = 1$, where x, y, u, v are real, then show that $\tanh^2 y \cosh^2 v = \sin^2 u$. **8**

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4. (a) If $ux + vy = a$, $\frac{u}{x} + \frac{v}{y} = 1$, Show that $\frac{u}{x}\left(\frac{\partial x}{\partial u}\right)_v + \frac{v}{y}\left(\frac{\partial y}{\partial v}\right) \cancel{= 0}$ 4 **6**

(b) If $(1 + i\tan\alpha)^{(1 + i\tan\beta)}$ is real then one of the principal values is $(\sec\alpha)^{\sec^2\beta}$. **6**

(c) Solve by Crout's Method the system of equaitons $2x + 3y + z = -1$, $5x + y + z = 9$, $3x + 2y + 4z = 11$ **8**

$b\cos 3\theta$

5. (a) If $\sin^4\theta\cos^3\theta = a\cos\theta + b\cos^3\theta + c\cos 5\theta + d\cos 7\theta$ then find a, b, c, d. **6**

(b) Use Taylor theorem and arrange the equaiton in powers of x. **6**

$$7 + (x+2) + 3(x+2)^3 + (x+2)^4 - (x+2)^5$$

(c) If $y = \cos(m\sin^{-1}x)$ prove that $(1 - x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} + (m^2 - n^2)\,y_n = 0$. **8**

6. (a) Solve correctly upto three iterations the following equations by Gauss-Seidel method. **6**
$10x - 5y - 2z = 3$, $4x - 10y + 3z = -3$, $x + 6y + 10z = -3$.

(b) If $u = \sin(x^2 + y^2)$ and $a^2x^2 + b^2y^2 = c^2$ find $\frac{du}{dx}$. **6**

(c) Fit a curve $y = ax + bx^2$ for the data : **8**

x	1	2	3	4	5	6
y	2·51	5·82	9·93	14·84	20·55	27·06

FE (SEM I) (RW) (CBSGS) May 2014
applied mathematics I 13/05/14

(REVISED COURSE) Q P Code : **NP-17690**

(3 Hours) [Total Marks : 80

N.B. : (1) Questions **No. 1** is **compulsory.**
(2) **Attempt** any **three** from the remaining questions.
(3) Assume suitable data if necessary.

1. (a) Prove that $\operatorname{Sech}^{-1}(\sin\theta)=\log\left(\cot\frac{\theta}{2}\right)$ 3

(b) If $x=\cos\theta - r\sin\theta$, $y=\sin\theta + r\cos\theta$ 3

prove that $\frac{dr}{dx}=\frac{x}{r}$

(c) If $x=e^{v}\sec u$, $y=e^{v}\tan u$ 3

find $J\left(\frac{u,v}{x,y}\right)$

(d) If $y=\sin px+\cos px$ 3

Prove that $y_n=p^n\left[1+(-1)^n\sin)2px)\right]^{\frac{1}{2}}$

(e) Find the series expansion of log (1+x) in powers of x. Hence prove that 4

$\log x=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3$........

(f) If 'A' is skew-symmetric matrix of odd order then prove that it is singular. 4

2. (a) Show that the roots of the equation $(x+1)^6+(x-1)^6=0$ are given by 6

$-i\cot\left(\frac{2n+1}{12}\right)\pi, n=0,1,2,3,4,5.$

(b) Find two non-singular matrices P & Q such that PAQ is in normal form where 6

$$A=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 1 & 4 & -5\\ -1 & -5 & -5 & 7\end{bmatrix}$$

Also find rank of A.

(c) If $x+y=2e^{\theta}\cos\phi$, $x-y=2ie^{\theta}\sin\phi$ & u is a function of x & y the prove that 8

$$\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\varphi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$$

3. (a) Find the value of λ for which the equations $x_1+2x_2+x_3=3$, $x_1+x_2+x_3=\lambda$, $3x_1+x_2+3x_3=\lambda^2$ has a solution & solve them completely for each value of λ. **6**

(b) Divide 24 into three parts such that the product of the first, square of the second & cube of the third is maximum. **6**

(c) (i) If $\operatorname{cosec}\left(\frac{\pi}{4}+ix\right)=u+iv$ prove that $(u^2+v^2)^2=2(u^2-v^2)$ **4**

(ii) Prove that $\tan\left(i\log\left(\frac{a-ib}{a+ib}\right)\right)=\frac{2ab}{a^2-b^2}$ **4**

4. (a) Show that $\frac{\partial(u,v)}{\partial(x,y)}=6r^3\sin 2\theta$ given that $u=x^2-y^2$, $v=2x^2-y^2$ & $x=r\cos\theta$, $y=r\sin\theta$. **6**

(b) If $\alpha=1+i$, $\beta=1-i$ & $\cot\theta=x+1$ prove that $(x+\alpha)^n+(x+\beta)^n=(\alpha+\beta)\cos n\theta\operatorname{cosec}^n\theta$. **6**

(c) Using Gauss-seidel method, solve the following system of equations upto 3rd iteration. **8**

$5x-y=9$
$-x+5y-z=4$
$-y+5z=-6$

5. (a) Using De-Moivr's theorem, prove that **6**

$$\frac{\sin 6\theta}{\sin\theta}=16\cos^4\theta-16\cos^2\theta+3$$

(b) Expland $\frac{x}{e^x-1}$ in powers of x. **6**

Hence prove that $\frac{x}{2}\left[\frac{e^x+1}{e^x-1}\right]=1+\frac{1}{12}x^2-\frac{1}{720}x^4+\ldots$

(c) If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ **8**

prove that $(1-x^2)y_{n+2}-(2n+3)xy_{n+1}-(n+1)^2y_n=0$. Hence find $y_n(0)$

6. (a) Examine the linear dependence or independence of vectors (1, 2, -1, 0), (1, 3, 1, 3), (4, 2, 1, -1) & (6, 1, 0, -5) 6

(b) If $u = f\left(\frac{x-y}{xy}, \frac{z-x}{xz}\right)$ prove that 6

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

(c) (i) Fit a straight line to the following data with x - as independent variable. 4

X :	1965	1966	1967	1968	1969
Y :	125	140	165	195	200

(ii) Evaluate $\lim\limits_{x \to 0} (1+\tan x)^{\cot x}$ 4

F.E Sem I Rev (CGS) 11/5/2013

Applied maths - I

D : PH (April Exam) 181

Con. 6865-13. **(REVISED COURSE)** **GS-5103**

(3 Hours) [Total Marks : 80

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **three** questions from Question Nos. 2 to Questions No. 6
(3) **Figures** to the **right** indicate **full** marks.

1. (a) If $\cos hx = \sec\theta$ prove that $x = \log(\sec\theta + \tan\theta)$. 3

(b) If $u = \log(x^2 + y^2)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ 3

(c) If $x = r\cos\theta$, $y = r\sin\theta$. Find $\frac{\partial(x, y)}{\partial(r, \theta)}$. 3

(d) Expand $\log(1 + x + x^2 + x^3)$ in powers of x upto x^8. 3

(e) Show that every square matrix can be uniquelly expressed as sum of a symmetric and a Skew-symmetric matrix. 4

(f) Find n^{th} order derivative of 4

$y = \cos x.\ \cos 2x.\ \cos 3x.$

2. (a) Solve the equation $x^6 - i = 0$. 6

(b) Reduce matrix A to normal form and find its rank where :– 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

(c) State and prove Euler's theorem for a homogeneous function in two variables and hence find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \frac{\sqrt{x} + \sqrt{y}}{x + y}$ 8

3. (a) Determine the values of λ so that the equations $x + y + z = 1$; $x + 2y + 4z = \lambda$; $x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case. 6

(b) Find the stationary values of 6

$x^3 + y^3 - 3axy$, $a > 0$.

(c) Separate into real and imagianary parts $\tan^{-1}(e^{i\theta})$. 8

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4. (a) If $x = u \cos v$, $y = u \sin v$ 6

Prove that $\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1$.

(b) If $\tan[\log(x+iy)] = a + ib$, prove that $\tan[\log(x^2+y^2)] = \frac{2a}{(1-a^2-b^2)}$ where 6

$a^2 + b^2 \neq 1$.

(c) Using Gauss-Siedel iteration method, solve 8

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14$$

upto three iterations.

5. (a) Expand $\sin^7\theta$ in a series of sines of multiples of θ. 6

(b) Evaluate $\lim_{x \to 0} \frac{(x^x - x)}{(x - 1 - \log x)}$ 6

(c) If $y^{1/m} + y^{-1/m} = 2x$, prove that 8

$$(x^2 - 1)\, y_{n+2} + (2n+1)\, x y_{n+1} + (n^2 - m^2)\, y_n = 0.$$

6. (a) Examine the following vectors for linear dependence/Independence. 6

$X_1 = (a, b, c)$, $X_2 = (b, c, a)$, $X_3 = (c, a, b)$ where $a + b + c \neq 0$.

(b) If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that 6

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

(c) Fit a straight line to the following data and estimate the production in the year 1957. 8

Year :	1951	1961	1971	1981	1991
Production in the Thousand tons :	10	12	08	10	13
